so that the lines of force at the disk are normal to it, but the direction of the force is reversed as we cross the axis of y. The current-function relatively to axes displaced through the proper angle η in the direction of rotation, varies as

$$y\sqrt{1-r^2/a^2}$$
.

A drawing of the current lines for this case is given. As already mentioned, they are simply the orthogonal projections of the contour lines of the tessaral harmonic of the second order.

In the next type we have n = 3, s = 2, so that

$$\bar{\Omega} \propto z(x^2-y^2),$$

and the current-function, relatively to displaced axes as before, varies as

$$xy \sqrt{1-r^2/a^2}$$
.

IV. "On the Magnetisation of Iron in Strong Fields." By Professor J. A. EWING, B.Sc., F.R.S.E., University College, Dundee, and Mr. WILLIAM LOW. Communicated by Sir W. THOMSON, Knt., LL.D., F.R.S. Received March 2, 1887.

(PLATE 2.)

The behaviour of iron and steel when subjected to very strong magnetising forces is a matter of considerable practical and very great theoretical interest, especially from its bearing on the molecular theory of magnetisation, which assigns an upper limit to the intensity of magnetism that a piece of iron can acquire, and even suggests that the metal may become diamagnetic under the influence of a sufficiently great force. All experiments hitherto made, by magnetising iron in the field of an electric solenoid, have shown that the intensity of magnetism J, as well as the induction J, is increasing with the highest values actually given to the magnetising force J. It is scarcely practicable, however, to produce by the direct action of a magnetising solenoid, a field whose force exceeds a few hundreds of C.G.S. units.

To refer to a few recent experiments of this class:—In experiments by one of us* on the magnetisation of long wires, the highest value of § applied to iron was about 90, and this gave an induction § of 16,500 in a soft iron wire. In Dr. Hopkinson's experiments† a force

^{*} Ewing, "Exp. Res. in Magnetism," 'Phil. Trans.,' 1885, Part II.

[†] J. Hopkinson, "Magnetisation of Iron," 'Phil. Trans.,' 1885, Part II.

of 240 gave 19,840 for the induction in a bar of mild Whitworth steel, and 18,250 in a bar of wrought iron.* The corresponding values of 3 are 1563 and 1437 respectively. Probably the highest magnetisation reached in any experiments of this class already published is that found by Mr. Shelford Bidwell† in his experiments on the tractive force between the halves of a divided ring electro-magnet. For a force 3 of 585 he gives 19,820 as the value of 3 (calculated from the tractive force) in a wrought-iron ring. The corresponding value of 3 is 1530.

With cast iron, Dr. Hopkinson found (in a sample of grey iron) 10,783 for the induction produced by a force of 240. The corresponding value of § is 841.

In the space between the pole-pieces of a strong electro-magnet we have a field of force of much greater intensity than it is practicable to produce by the direct action of the electric current. This field is not well adapted for experiments whose object is to determine with precision the relation of magnetisation to magnetising force, on account of the distortion which it undergoes when the piece of iron to be magnetised is introduced into it. It is, however, well suited for experiments whose object is to determine how much magnetism the metal can be forced to take up.

For this purpose it is of course necessary that the cross-section of the test-piece should be much smaller than the area of the pole-piece faces. In the following experiments the electro-magnet consisted of a pair of vertical limbs 25 cm. long, with cores 5 cm. in diameter, joined at the bottom by a horizontal yoke, and furnished on the top with pole-pieces, made of soft hammered scrap iron, in the form of rectangular blocks with plane faces, whose distance from each other could be adjusted at will. The faces were 5.25 cm. square. The magnet was wound with wire large enough to permit a current of about 27 ampères to be used for a short time. In the earliest experiments the test-piece to be magnetised was a round cylinder of soft iron, with flat ends 0.34 cm. in diameter and 1.3 cm. long. This was covered with an induction coil, consisting of a single layer of fine wire, which extended over the whole length of the piece. It was placed lengthwise in the centre of the field, with the pole-pieces just touching its ends, and the field magnet was excited. The test-piece was then suddenly withdrawn, while the transient current produced in the induction coil was measured by a ballistic galvanometer connected to the induction coil by long leading wires, which were twisted together

^{*} J. and E. Hopkinson have observed an induction of 20,000 in the core of a dynamo-armature, under a force estimated at 740 ('Phil. Trans.,' 1886 (Part I) n. 355).

⁺ S. Bidwell, "On the Lifting Power of Electro-magnets and the Magnetisation of Iron," 'Roy. Soc. Proc.,' vol. 40, 1886, p. 486.

throughout their whole length. Very few experiments were made with test-pieces of this form, for it was found that they gave by no means an exceptionally high value for the magnetic induction. This is to be ascribed to the fact that the ends of the cylinder, which were in contact with the pole-pieces, necessarily shared that value of the induction which existed in the part of the pole-piece faces which they touched, and this comparatively low induction in and near the ends of the cylinder neutralised the much higher value in the middle portion. The induction coil, being wound from end to end of the bar, gave a mean value for the whole length. To obtain higher values, it was obviously necessary to restrict the measurement of the induction to the middle portion, where the induction was greatest; and, further, it was desirable to furnish the bar with conical or some form of spreading ends, which would present an easy path for the lines of induction to converge towards the central neck. Accordingly, test-pieces were turned of the form and dimensions of Sample A. shown in Plate 2, fig. 1, where the bobbin is sketched in place between the pole-pieces. These were wound along the whole length of the narrow central neck with an induction coil consisting of a single layer of No. 36 S.W.G. silk-covered wire. In Sample A the diameter of the iron neck was 0.923 mm., and the diameter measured to the middle of the thickness of the wire forming the induction coil was 0.9495. Hence there was but little space, outside the section of the iron, enclosed by the coil; and the small amount of magnetic induction in this non-ferrous space was allowed for by a method to be explained below.

In test-pieces of the form of Sample A the loss of magnetism observed on suddenly withdrawing the piece from its place between the pole-pieces of the field magnet, is less than the whole magnetism by the small but somewhat uncertain quantity of residual magnetism which the piece retains. To avoid this source of uncertainty another form of test-piece was used, which is shown in fig. 2. Sample B. Here the bobbin has its conical ends rounded at the base to form portions of a circular cylinder, and the pole-pieces are hollowed to correspond. The bobbin can now be turned completely round about a central axis at right angles to the paper, so that the direction of its magnetism is reversed, and half the ballistic effect of the reversal measures the magnetic induction. This method was used in the greater number of the observations. Again, by merely withdrawing the bobbin from the field, and comparing the effect of this withdrawal with half the effect of reversal, an estimate was arrived at of the amount of error to which the former experiments were subject on account of residual magnetism.

To determine the intensity of the magnetic field in the space immediately surrounding the narrow neck in which the greatest induction occurred, a small quantity of wire was wound over the first induction coil, to form a distance-piece, and on the top of that a second induction coil was wound, the second coil, like the first, consisting of a single layer of very fine wire. The space between the two coils was accurately determined. When the test-piece was reversed or drawn out of the field the operation was in each case performed several times, and two groups of observations were recorded, one giving the induction in the inner coil, and the other the induction in the outer coil; the difference of course served to determine the field in the space between the coils. When this field was known it was easy to correct for the induction in the non-ferrous space enclosed by the inner coil.

Three kinds of wrought iron were tested; soft hammered scrap, Swedish iron, and Lowmoor iron. The hammered scrap proved less susceptible than the other two, and was not used in the final experiments, which were made with test pieces of the form of Sample B. Pieces of cast iron were also tested, in forms resembling both A and B.

To determine in absolute measure the value of the ballistic effects, a large earth-coil was kept in circuit with the induction coil and galvanometer, and was turned over in either the vertical or horizontal earth-field at the beginning, and again at the end of each group of observations. To avoid the possibility of error in this important particular, two separate earth-coils of entirely different dimensions were employed, and the galvanometer constant was determined independently by means of both, with results which were in excellent agreement. The values of the induction stated below are worked out on the basis that the horizontal force in the grounds of University College, Dundee, at a place sufficiently removed from local magnetic influence, is 0·160 in C.G.S. units.

The following experiments are representative of a considerably larger number:—

Lowmoor iron, annealed before turning the bobbin from a forged bar. Sample B, of shape and dimensions shown in fig. 2. Diameter of iron neck = 0.65 cm.; length = 0.44 cm. Diameter to middle of inner induction coil, 0.6765 cm. Diameter to middle of outer induction coil, 0.9364 cm.

Area of section of iron $(S_1) = 0.3318$ sq. cm.

Area of space to be corrected for under inner induction coil $(S_2) = 0.0276$ sq. cm.

Area of space between inner and outer coil (S_3) = 0.3293 sq. cm.

Number of turns on inner induction coil = 16; number on outer coil = 12.

In the following table D_1 is the throw of the ballistic galvanometer given by the inner coil when the test-piece was turned round, and D_2 is the throw given by the outer coil. X_1 and X_2 are the corre-

26.8

31,560

10,780

166

14,460

165

sponding total inductions in C.G.S. units. The difference of these, given in the fifth column, when divided by S_3 , is the intensity of field or magnetic force per sq. cm., in the space immediately surrounding the iron. This is given in column VI. Multiplying it by S_2 , we have the correction to be subtracted from X_1 , which is given in column VII. Finally, by dividing the corrected value of X_1 by the section of the iron S_1 , we find \mathfrak{B} , the magnetic induction in the iron per sq. cm. Column IX gives the current in the field magnet coils in ampères.

I.	II.	III.	IV.	v.	VI.	VII.	VIII.	IX.
D ₁ .	X_1 .	D ₂ .	\mathbf{X}_2 .	X ₂ -X ₁ .	Field round iron neck per sq. cm.	Correction to be subtracted from X_1 .	% .	Current in field magnets, ampères.
127	8,295	109	9,490	1195	3,630	100	24,700	1.98
143	9,340	$132\frac{1}{2}$	11,540	2200	6,680	1.80	27,610	4.04
150	9,800	142	12,370	2570	7,800	220	28,870	5 .81
153	9,990	148	12,890	2900	8,810	250	29,350	7.60
$157\frac{1}{2}$	10,280	154	13,410	3130	9,500	260	30,200	11.0
160	10,450	157	13,670	3220	9,780	270	30,680	13.5
161	10,520	160	13,930	3410	10,360	290	30,830	16.2
164	10,710	164	14,280	3570	10,840	300	31,370	21.6
'	1	1 1		1	1	1 1		1

Lowmoor Wrought Iron: Sample B.

In another test of Lowmoor iron, conducted in the same way, a still higher value of 30 was reached, namely, 32,880. This is the highest induction that has been recorded in these experiments.

3680

11,180

310

A similar experiment with a piece of Swedish wrought iron, of the form and dimensions shown in fig. 2, gave 32,310 for the greatest value of \mathfrak{B} , the magnetic force in the ring of space surrounding the iron neck being then 11,250.

The amount of residual magnetism retained by a Lowmoor sample of this form (Sample B) was determined by comparing the effect of withdrawing the test-piece with the effect of reversing it. The results showed that within the range of magnetic force used in these experiments, namely, from about 4000 to 11,000 C.G.S. units, the residual magnetism is nearly constant. Its mean value in a number of determinations was—

For Lowmoor iron, residual induction, $\Re_r = 510$ per sq. cm.

For Swedish iron, residual induction, $\mathfrak{B}_r = 500$ per sq. cm.

These results showed that pieces of the form of Sample B (fig. 2) retained only a small part (less than 1/60) of their greatest induction when withdrawn from the field. The proportion of residual to

greatest induced magnetism in samples of the form A (fig. 1), is probably not very different from this.

In the following experiment a bobbin of annealed Swedish iron, of the size and shape shown in fig. 2, was tested by withdrawing it from the field. The columns of the table have the same meaning as before, except that the quantity in column VIII, now headed $\mathfrak{F} - \mathfrak{F}_r$, is not the whole induction per sq. cm., but that part of the induction which disappeared when the test-piece was withdrawn from the field. In this case the section of the iron was the same as before, but the space between the inner and outer induction coils (S₃) was 0.308 sq. cm. There were fourteen turns in the inner coil and twelve in the outer.

Swedish	Wrought	Iron:	Sample	В.

I.	II.	III.	ıv,	v.	VI.	VII.	VIII.	IX.
D ₁ .	$\mathbf{X}_{\mathbf{l}^{\star}}$	D ₂ .	\mathbf{X}_2 .	X_2-X_1 .	Field round iron neck per sq. cm.	Correction to be subtracted from X_1 .	$\mathbb{B}-\mathbb{B}_{r}$.	Current in field magnets, ampères.
125.5	9,290	131.5	11,350	2060	6,690	180	27,460	4.08
134.5	9,950	147.0	12,690	2740	8,900	250	29,230	7.77
139.5	10,320	153.5	13,250	2930	9,510	260	30,320	10.9
141 .5	10,470	157 .0	13,550	3080	10,000	280	30,710	14.2
143 .5	10,620	160.0	13,810	3190	10,360	290	31,130	16.5
144.0	10,660	162.0	13,990	3330	10,810	300	31,220	18.9
145.5	10,770	163 .5	14,120	3350	10,880	300	31,560	22.9
147 .0	10,880	166.0	14,330	3450	11,200	310	31,860	26.5
						1		

The residual magnetism may be corrected for by adding 500 as the value of \mathfrak{B}_r to each of the numbers in column VIII. We then obtain for the highest induction \mathfrak{B} the value 32,360.

The following results relate to test-pieces of the form and size shown in fig. 1:—

Swedish wrought iron. Form of Sample A. Section of iron neck = 0.669 sq. cm. Section to middle of induction coil = 0.708 sq. cm. Loss of induction per sq. cm. tested on withdrawing the bobbin $(\mathcal{B} - \mathcal{B}_r)$.

Current in field magnets,	$\mathfrak{B}-\mathfrak{B}_{r}$	
ampères.		
3.92	$27,\!550$	
7·4 8	29,420	
11:3	30,240	
14:0	30,460	
17.9	30,960	
20.1	31,180	
20 4	31,290	

These figures agree very well with those in the preceding table, which related to another sample of different form cut from the same bar. Probably 500 is in this case also a fair estimate of the residual induction, and by adding that to the values given above we arrive at probable values of **3**.

Lowmoor wrought iron. Form of Sample A. Dimensions as above, Current in field magnets = 20.4 ampères. Loss of induction on withdrawing the bobbin $(\mathfrak{B} - \mathfrak{B}_r) = 31,660$. Allowing for the residual magnetism, this gives an induction exceeding 32,000.

Soft Hammered Scrap. Form of Sample A. Dimensions as above.

Current in field magnets.	$\mathfrak{B}-\mathfrak{B}_{r}$.
20.4	31,230
26.2	31,520

The remaining experiments relate to cast iron. The following results are for a sample of the form shown in fig. 2, except that the neck was of considerably larger diameter, namely 0.962 cm. The sample was tested by turning it end for end in the magnetic field.

Section of neck = 0.727 sq. cm.

Section within middle of inner induction coil = 0.767 sq. cm.

Space to be corrected for = 0.040 sq. cm.

Section within middle of outer induction coil = 1.195 sq. cm.

Space between coils = 0.328 sq. cm.

Cast Iron.

X ₁ .	X_2 .	X_1-X_2 .	Field round iron neck per sq. cm.	Correction to be subtracted from X ₁ .	X .	Current in field magnets, ampères.
14,450	15,730	1280	3,900	160	19,660	1·97
16,200	18,300	2100	6,400	260	21,930	3·75
16,910	19,440	2530	7,710	310	22,830	5·38
17,420	20,070	2650	8,080	320	23,520	7·08
18,240	21,260	3020	9,210	370	24,580	13·15
18,490	21,670	3180	9,700	390	24,900	16·9
19,030	22,510	3480	10,610	420	25,600	22·6

Another set of readings were taken with this sample at the same time, by drawing it suddenly out of the field, in order to determine the residual induction. The results showed that throughout the range of magnetic forces employed here, the residual induction had a nearly constant value of 400 C.G.S. units per sq. cm.

A bobbin of cast iron of a form resembling Sample A, fig. 1, was

also tested by drawing it out of the field. The results were in close agreement with those given above for the other sample.

In fig. 3 the general results for Lowmoor wrought iron (Sample B) and cast iron are shown by curves which give the relation (1) of the induction \mathfrak{B} within the metal neck to the current in the field magnet coils, and (2) of the induction or magnetic force in the space immediately surrounding the neck to the current in the field magnet coils. The full lines are for the Lowmoor forging, and the broken lines are for cast iron. The field produced by a given current is (at its higher values) rather less strong in the case of cast iron, probably because the larger size of the cast iron neck allowed a greater portion of the whole induction from pole to pole to find its way through the metal. (Compare X_1 for cast iron and for Lowmoor.)

The magnetic force within the metal (3) differs from the field in the surrounding space by an amount which cannot be estimated without a knowledge of the distribution of free magnetism on the pole-pieces and conical faces of the bobbin. It appears probable that with the dimensions of the various parts used in these experiments. the magnetic force within the metal is less, but not very greatly less, than the outside and closely neighbouring field. In the absence of any exact knowledge of M, it is interesting to examine the relation of B to the outside field. Thus, (B-outside field)/4m gives a quantity which is probably not much less than the intensity of magnetism J. The values of this quantity for Lowmoor wrought iron, Swedish wrought iron, and cast iron are stated below. In the case of the Swedish iron the values of B-B, given in the previous table for that metal have had 500 added to allow for the residual magnetism. Again, the quantity Bootside field is probably not much less than the magnetic permeability μ : its values also are given below.

I. Lowmoor Wrought Iron.

Outside field.	B .	8 – outside field 4π .	B outside field.	
3,630	24,700	1680	6.80	
6,680	27,610	1670	4.13	
7,800	28,870	1680	3·7 0	
8,810	29,350	1630	$3 \cdot 33$	
9,500	30,200	1650	3 · 18	
9,780	30,680	1660	3.14	
10,360	30,830	1630	2.98	
10,840	31,370	1630	2 .89	
11,180	31,560	1620	2 · 82	

Outside field.	38.	$\frac{8 - \text{outside field}}{4\pi}$	% outside field.	
6,690	27,960	1700	4·18	
8,900	29,730	1660	3·34	
9,510	30,820	1700	3·24	
10,000	31,210	1690	3·12	
10,360	31,630	1700	$egin{array}{c} 3 \cdot 05 \\ 2 \cdot 94 \\ 2 \cdot 95 \\ 2 \cdot 90 \end{array}$	
10,810	31,720	1670		
10,880	32,060	1690		
11,200	32,360	1690		

II. Swedish Wrought Iron.

III. Cast Iron.

Outside field.	¥.	$\frac{\mathfrak{B}-\text{outside field.}}{4\pi}$	outside field.
3,900	19,660	1250	5·04
6,400	21,930	1240	3·42
7,710	22,830	1200	2·96
8,080	23,520	1230	2·91
9,210	24,580	1220	2·67
9,700	24,900	1210	2·57
10,610	25,600	1190	2·46

Fig. 4 shows by curves the relation of \mathfrak{B} to \mathfrak{B} /outside field for Lowmoor iron and for cast iron, in the manner introduced by Rowland for showing the relation of \mathfrak{B} to μ . The curves have the same kind of inflection that a curve of μ and \mathfrak{B} begins to have when the magnetising force is raised sufficiently high.* The range through which the permeability of iron may vary is well shown by comparing the values reached here (probably in the extreme case less than 3) with the value 20,000, which was found by one of us in the case of a soft wire exposed to a very small magnetising force and kept at the same time in a state of mechanical vibration.†

The quantity (\mathfrak{F} —outside field)/ 4π is nearly constant in the Swedish iron, but diminishes with increased induction in the Lowmoor iron and in the cast iron. If the outside field were an accurate measure of

^{*} This feature of the curve of μ and % was not noticed by Rowland himself, who applied to his curve an empirical formula which fails to take account of it. It has, however, been noticed by several later observers (Fromme, 'Wiedemann, Annalen,' vol. 13, p. 695; Ewing, loc. cit., p. 574; Bidwell, loc. cit., p. 495).

[†] Ewing, loc. cit., p. 567.

If, this would mean that in the two metals last named I had passed a maximum, and the process of diamagnetisation which the Ampère-Weber molecular theory of magnetism anticipates had set in. But the uncertainty which attaches to the value of I prevents this conclusion from being fairly drawn from these experiments. A slight excess in the mean value of n within the metal neck over the value of n in the space contiguous to the neck would suffice to convert the apparent decrease of I into an increase, with increasing values of B. these results can be said to bear upon the point in question, they rather support the idea that the intensity of magnetism & becomes and remains a sensibly constant quantity when the magnetising force is raised to very high values. This maximum of I appears to exceed 1700 in wrought iron and 1250 in cast iron, and it does not appear likely that any increase of magnetising force will bring the intensity of magnetism in cast iron to a value equal or nearly equal to that which wrought iron is capable of acquiring. It is scarcely necessary to add that our experiments give no support to the suggestion that there is a maximum of the induction B. The value of B capable of being reached by the method we have employed depends mainly on the scale of the experiments. Larger field magnets with pole-pieces tapering to a narrow neck should yield values of B greatly in excess even of those we have observed.

The experiments will be continued and various qualities of steel will be examined with the following modification in the apparatus:—The pole-pieces will themselves be turned, at the ends which face each other, into cones with flat ends, between which the test-piece in the form of a round cylinder will be inserted. The induction will be measured in the neighbourhood of a medial transverse plane only, and the value of the field outside the iron will be determined in this plane at various distances from the axis. Since there is no free magnetism in the iron bar in the medial plane, the magnetic force within the metal is continuous with the force in the surrounding space, and a curve showing the relation of the magnetic force at various points outside to the distance from the axis should admit of being produced so as to give a good approximation to the magnetic force within the metal. If this can be successfully accomplished, the value of the isthmus method of examining the magnetisation of iron will be greatly enhanced.

[Dr. Hopkinson informs me that he experimented by what we have called the "isthmus" method nearly three years ago, but gave it up from uncertainty about the induction which took place through the coil but not through the iron. In the present experiments this difficulty has been avoided mainly by using larger bobbins with a single layer of fine wire for induction coil. I am indebted to Dr. Hopkinson for the suggestion (soon to be put in practice) that the "isthmus"

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method should be applied to the manganese steel whose non-magnetic quality under ordinary conditions has been already commented on by himself as well as by Mr. J. T. Bottomley and Professor Barrett. In connexion with the values of B reached by other observers, Professor J. J. Thomson informs me that in some recent experiments by himself and Mr. H. F. Newall on the effect of cutting a magnet at right angles to the lines of force, an induction of 28,000 was found on one occasion.—J. A. E.]

Presents, March 24, 1887.

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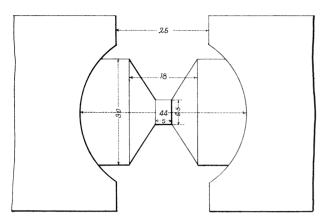
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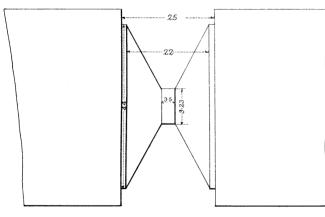
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Fig. 2.



SAMPLE IN POSITION.
B.

Fig.1.



SAMPLE IN POSITION. A.

